Homogeneous Differential Equations

See page 373

Examples

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$f(x,y) = x^3 + 3x^2y^2 - 2y^2$

$f(tx, ty) = t^3x^3 + 3t^2x^2t^2y^2 - 2t^2y^2$

$= t^3x^3 + 3t^4x^2y^2 - 2t^2y^2$

Not Homogeneous

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$f(x,y) = \dfrac{xy}{\sqrt{x^2+y^2}}$

$f(tx,ty) = \dfrac{t^2xy}{\sqrt{t^2(x^2+y^2)}} = \dfrac{t^2xy}{t\sqrt{x^2+y^2}} = t\left[\dfrac{xy}{\sqrt{x^2+y^2}}\right]$

$= tf(x,y) \rightarrow$ Homogeneous of degree 1.

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$$y' = \frac{x^3+y^3}{xy^2}$$

$$xy^2\,dy = (x^3+y^3)\,dx$$

$y = vx$

$dy = x\,dv + v$

$$x(vx)^2(x\,dv + v\,dx) = (x^3 + v^3x^3)\,dx$$

$$x^4v^2\,dv + x^3v^3\,dx = x^3\,dv + v^3x^3\,dx$$

$$x^4v^2\,dv = x^3\,dv$$

$$\int v^2\,dv = \int \frac{1}{x}\,dx$$

$$\frac{v^3}{3} = \ln|x| + C_1$$

$$v^3 = 3\ln|x| + C$$

$$\left(\frac{y}{x}\right)^3 = 3\ln|x| + C$$

$$y^3 = 3x^3\ln|x| + Cx^3$$

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$y = vx \qquad dy = x\,dv + v\,dx$

$(2x^2+y^2)\,dx + xy\,dy = 0 \qquad y(1) = 0$

$(2x^2 + v^2x^2)\,dx + vx^2(x\,dv + v\,dx) = 0$

$2x^2\,dx + v^2x^2\,dx + vx^3\,dv + x^2v^2\,dx = 0$

$(2x^2 + 2x^2v^2)\,dx = -vx^3\,dv$

$(2+2v^2)\,dx = -vx\,dv$

$\int \frac{-2}{x}\,dx = \int \frac{v}{1+v^2}\,dv$

$-2\int \frac{1}{x}\,dx = \frac{1}{2}\int \frac{2v}{1+v^2}\,dv$

$-2\ln|x| = \frac{1}{2}\ln|1+v^2| + C_1$

$\ln x^{-2} = \ln\sqrt{1+v^2} + \ln C$

$\ln x^{-2} = \ln C\sqrt{1+v^2}$

$x^{-2} = C\sqrt{1+v^2}$

$\frac{1}{x^2} = C\sqrt{1+\frac{y^2}{x^2}} = C\sqrt{\frac{x^2+y^2}{x^2}}$

$\frac{1}{x^2} = \frac{C}{x}\sqrt{x^2+y^2}$

$$\frac{1}{x} = C\sqrt{x^2+y^2} \qquad y(1)=0$$

$$\frac{1}{1} = C\sqrt{1^2+0^2}$$

$$C = 1$$

$$\frac{1}{x} = \sqrt{x^2+y^2}$$ Particular Solution

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