

Chapter 5 problems

SHOW ALL OF YOUR WORK AND CIRCLE YOUR ANSWERS.

1. Find the derivative: $f(x) = \ln(x^3 + 3x)^3$.

2. Differentiate: $y = \ln(\ln \tan x)$.

3. Use a graphing utility to graph the function $f(x) = \ln \frac{1}{x^2 + x - 6}$.

a. Use the graph to evaluate the limit: $\lim_{x \rightarrow 2^+} \left[\ln \frac{1}{x^2 + x - 6} \right]$.

b. Use the graph to evaluate the limit: $\lim_{x \rightarrow +\infty} \left[\ln \frac{1}{x^2 + x - 6} \right]$.

c. Use the graph to state the domain of f .

4. Evaluate the integral: $\int_2^{e+1} \frac{1}{x-1} dx$.

5. Evaluate the integral: $\int \frac{x^2 + x + 1}{x^2 + 1} dx$.

6. Evaluate the integral: $\int \frac{1 - \sin \theta}{\cos \theta} d\theta$.

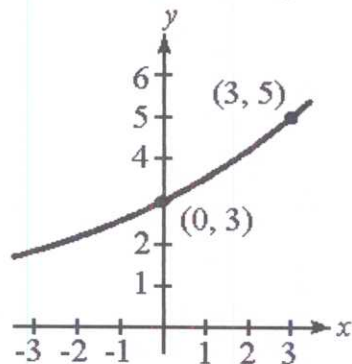
7. Given $f(x) = 2x^2 + 1$ for $x \geq 0$, find $f^{-1}(x)$.

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8. Find $\frac{dy}{dx}$: $y = (e^x + e^{3x})^4$.
9. Evaluate the integral: $\int \sin x e^{\cos x} dx$.
10. Evaluate the integral: $\int \frac{e^{3/x}}{x^2} dx$.
11. Sketch the graph: $f(x) = 1 + \log_5 x$.
12. If an annual rate of salary increase averages 4.5% over the next 5 years, then the approximate salary S during any year in that period is
- $$S(t) = P(1.045)^t$$
- where t is the time in years and P is the present salary.
- If a person's salary is \$30,000 now, use the function S to estimate her salary 5 years from now.
 - Use the model given to estimate how long it will be before this individual earns \$50,000.
 - Find the rate of change of S with respect to t when $t = 1$ and when $t = 4$.
13. The population P of a city is given by $P = 2000e^{kt}$. Let $t = 0$ correspond to the year 1960 and suppose the population in 1950 was 1500. Find the value of k (to 3 decimal places) and then predict the population in 1990.
14. Write an equation for the amount Q of a radioactive substance with a half-life of 30 days, if 10 grams are present when $t = 0$.
15. The number of fruit flies increases according to the law of exponential growth. If initially there are 10 fruit flies and after 8 hours there are 30, find the number of fruit flies after t hours.

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16. Find the constant k so that the exponential function $y = 3e^{kt}$ passes through the points given on the graph.



17. Find the particular solution to the differential equation $\frac{dy}{dx} = y^2(1 + x^3)$ given the general solution $y = \frac{-4}{4x + x^4 + C}$ and the initial condition $y(0) = 5$.
18. Use integration to find a general solution to the differential equation $\frac{dy}{dx} = \frac{3x^3}{1 + x^2}$.
19. Find the general solution to the first-order differential equation: $xy \, dx - (x^2 + y^2) \, dy = 0$.
20. Find the general solution of the differential equation $\frac{y'}{x} = \frac{e^x}{y^2}$.
21. Find the particular solution of the differential equation $\frac{dy}{dx} = 500 - y$ that satisfies the initial condition $y(0) = 7$.
22. A colony of bacteria increases at a rate proportional to the number present. If there were 1000 bacteria present in the beginning of the experiment and the number triples in four hours, determine the number present as a function of time.

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23. Find the derivative: $y = \arctan e^x$.24. Use a graphing utility to graph the function $f(x) = \operatorname{arccot}(x - 1)$.25. Evaluate the integral: $\int \frac{5x + 16}{x^2 + 9} dx$.26. Evaluate the integral: $\int \frac{2 - x}{\sqrt{4 - x^2}} dx$.27. Evaluate the integral: $\int \frac{1}{x^2 + 4x + 7} dx$.28. Evaluate the integral: $\int \frac{1}{(x - 3)\sqrt{x^2 - 6x + 13}} dx$.29. Consider $f(x) = \frac{1}{4 + 9x^2}$.

- Use a graphing utility to graph f .
- Sketch the region bounded by f , the x -axis, and the lines $x = 1$ and $x = 2$.
- Write the integral that represents the area of the region.
- Calculate the area.

30. Use a graphing utility to graph the function $f(x) = \sinh \frac{x}{2}$. Then sketch the region bounded by f , the x -axis, and the line $x = 3$ and calculate its area.

Chapter 5 problems

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1.
$$\frac{9(x^2 + 1)}{x(x^2 + 3)}$$
 Chapter: 051 Question: 18

2.
$$\frac{\tan x[\ln(\tan x)]}{\sec^2 x}$$
 Chapter: 051 Question: 25

3. **a.** $+\infty$
b. $-\infty$
c. $(-\infty, -3), (2, \infty)$
 Chapter: 051 Question: 33

4. 1
 Chapter: 052 Question: 3

5.
$$x + \frac{1}{2} \ln(x^2 + 1) + C$$
 Chapter: 052 Question: 10

6. $\ln|1 + \sin \theta| + C$
 Chapter: 052 Question: 22

7.
$$f^{-1}(x) = \sqrt{\frac{x - 1}{2}}$$
 Chapter: 053 Question: 9

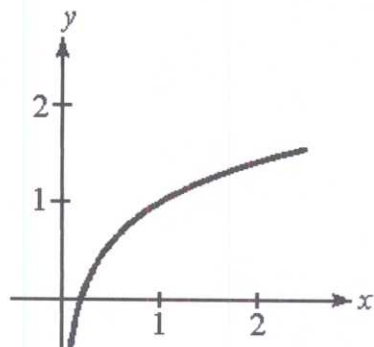
8. $4(e^x + 3e^{3x}) (e^x + e^{3x})^3$
 Chapter: 054 Question: 16

9. $-e^{\cos x} + C$
 Chapter: 054 Question: 21

10.
$$-\frac{e^{3/x}}{3} + C$$
 Chapter: 054 Question: 29

11. See graph below.
 Chapter: 055 Question: 15

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12. a. \$37,385.46

b. 11.6 years

c. 1379.93, 1574.73

Chapter: 055 Question: 19

13. $k = 0.029$; In 1990, $P = 4774$

Chapter: 056 Question: 16

14. $Q(t) = 10e^{-0.0231t}$

Chapter: 056 Question: 18

15. $y = 10e^{(\ln 3/8)t}$

Chapter: 056 Question: 23

16. $k = -\ln \frac{1}{3} - \frac{5}{3} \approx 0.1703$

Chapter: 056 Question: 15

17.
$$y = \frac{-4}{4x + x^4 - 4} = \frac{-20}{20x + 5x^4 - 4}$$

Chapter: 057 Question: 6

18.
$$y = -x - \frac{1}{2} \ln(1 + x^2) + C$$

Chapter: 057 Question: 11

19. $x^2 = y^2 \ln Cy^2$

Chapter: 057 Question: 18

20.
$$y^2 = e^x + C$$

Chapter: 057 Question: 22

21. $y = 500 - 493e^{-x}$

Chapter: 057 Question: 24

22. $y = 1000e^{(t \ln 3)/4}$

Chapter: 057 Question: 37

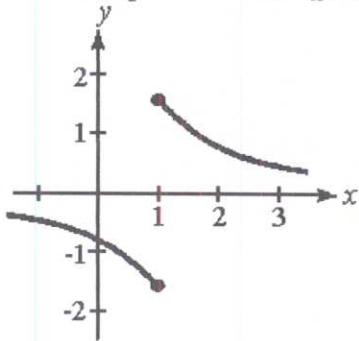
23.
$$\frac{1 + e^{2x}}{e^x}$$

Chapter: 058 Question: 20

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24. See graph below.

Chapter: 058 Question: 27



25.
$$\frac{5}{2} \ln(x^2 + 9) + \frac{16}{3} \arctan \frac{x}{3} + C$$

Chapter: 059 Question: 6

26.
$$2 \arcsin \frac{x}{2} + \sqrt{4 - x^2} + C$$

Chapter: 059 Question: 7

27.
$$\frac{1}{\sqrt{3}} \arctan \left[\frac{x+2}{\sqrt{3}} \right] + C$$

Chapter: 059 Question: 16

28.
$$\frac{1}{2} \operatorname{arcsec} \frac{x-3}{2} + C$$

Chapter: 059 Question: 19

29.

a. See graph below.

b. See graph below.

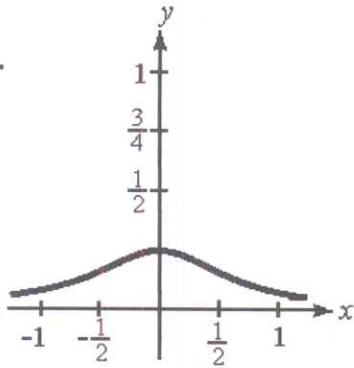
c.
$$\int_1^2 \frac{1}{4 + 9x^2} dx$$

d.
$$\frac{1}{6} \left[\arctan 3 - \arctan \frac{3}{2} \right] \approx 0.0444$$

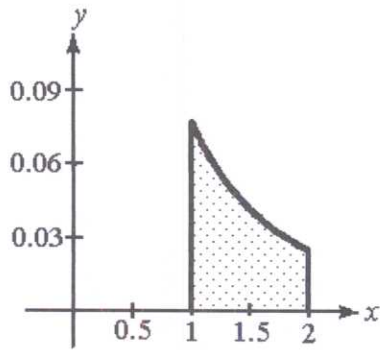
Chapter: 059 Question: 25

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a.



b.



30. See graph below.

$$2 \left[\cosh \frac{3}{2} - 1 \right] \approx 2.7048$$

Chapter: 510 Question: 20

