Algebra: Is Not Just Math / It's the Language of Science

Whether by coincidence or by design, Colman McCarthy kicked off Mathematics Awareness Week with an April 20 column suggesting that it is algebra that should be kicked off—off the list of courses required in our nation's schools.

He reasoned that teaching algebra to everyone will not address such problems as crime and drugs or pollution; that algebra has little to do with mathematics, and that he has never seen a help-wanted ad for an algebraist.

The temptation to the mathematician, after getting over the initial sputtering and grinding of chalk dust in one's teeth, is to respond with the kind of overstatement in defense of algebra that McCarthy has employed in his attack. The damage he has done is too serious, however, and calls for a more reasoned response.

Do the majority of adults use algebra? Of course not.

I think we can go further, and say that there are working engineers and other people with technical jobs who do not use algebra on a day-to-day basis.

McCarthy's physician may well have forgotten algebra; but that physician without doubt needs the background provided by his or her chemistry course, and one can't learn chemistry without first taking algebra.

That is the point to remember. Mathematics is not just another science; it is the language through which all of science and much of management science is taught.

The student who closes the door on high school algebra (and so on all of mathematics) closes the door on much more: all of engineering and science, the world of computer programming, anything that requires an understanding of statistics, electronics, medicine and medical technology, most management and MBA programs, and more. Make no mistake. The young person who drops out of algebra has dropped out of a lot more than he or she realizes.

Must algebra be difficult? Not really. Is it sometimes poorly taught? Certainly. Do some students get by with memorization instead of understanding? Of course. These are not arguments for abandoning algebra, however, but for teaching it better, a goal toward which renewed efforts of the National Council of Teachers of Mathematics are directed.

McCarthy is certainly right to worry about drugs and crime. We are not experts on the subject, but social scientists have led us to believe that a root cause is the inability of people to get jobs and the resultant poverty and sense of being left out. He is also right to identify pollution as part of the national mess; we do need cleaner sources of energy, and better handling of the waste we generate.

To whom will we turn for help? We don't know. We can't go into our grade schools and identify the problem solvers of tomorrow. We can say with reasonable assurance, however, that they won't come from the ranks of those who decided not to study algebra.

In all of this, we have focused on jobs and said nothing about the need for informed citizens to understand statistics as they face a medical choice, to have a feel for rate of growth when listening to economic forecasts, or to understand the concept of future value when they choose among retirement options.

Neither have we spoken of our need for teachers at all levels to know at least enough about the use of algebra so as to help their students do what McCarthy says he cannot: distinguish between the importance in our society of algebra and Sanskrit. In the best of all worlds, we could even hope for columnists who would not write such nonsense as "Algebra has little to do with mathematics."

Think of your own child, or a neighbor child, or the disadvantaged child in whom you have taken an interest. Do what you can to help that child experience a little more of what it means to have equal opportunity. Tell that young person to keep options open; to take algebra, and to follow it with geometry, and all the mathematics he or she can master; and to ignore Colman McCarthy. ■

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***(Editorial** continued from opposite page.)*

So how do we start to change the hopelessly misguided common perception of our subject? Certainly not by overstating our case. Nor by trying to make sure that everyone knows how to solve a quadratic or differentiate a polynomial. The first, urgent priority is to raise the public awareness of what mathematics is, its nature, and the scope of its applications. An excellent example of how to go about this is provided by the response to the McCarthy column written by Wayne Roberts, the Chair of the Department of Mathematics and Computer Science at Macalester College in St. Paul, Minnesota, which appeared in the *Star Tribune, Minneapolis* (though not, regrettably, in the *Washington Post*) a week after the original McCarthy piece. I think Roberts' letter deserves greater distribution than just one local newspaper. Of course, reproducing it in full, as I do in this month's FOCUS (see shaded box above), will not in itself further our cause; FOCUS simply preaches to the converted. But if FOCUS readers across the country took it upon themselves to use their local media to put across Roberts' message, then maybe we can start to make some progress.

Keith Devlin ■

The views expressed in this column represent those of the FOCUS editor. They do not necessarily represent the official position of the MAA.

Two examples may look the same, but the instructions may be asking you to do two different things. For example,

Identify the following property: $a + (b + c) = (a + b) + c$

versus

Verify the following property by replacing the variables with numbers:
$a + (b + c) = (a + b) + c$

On the other hand, two different examples may have the same instructions but require you to do different things. For example,

Evaluate: $2(3 + 8)$ versus Evaluate: $2 + (3 + 8)$

You are asked to evaluate both expressions, but the solutions require different steps.

It is a good idea to familiarize yourself with the various ways the same basic instructions can be worded. In any case, always look at an example carefully and ask yourself what is being asked and what needs to be done, *before you do it.*

Comparing and Contrasting Examples

When learning most things for the first time, it is very easy to get confused and treat things which are different as though they were the same because they "look" similar. Algebraic notation can be especially confusing because of the detail involved. Move or change one symbol in an expression and the entire example is different; change one word in a verbal problem and the whole problem has a new meaning.

It is important that you be capable of making these distinctions. The best way to do this is by comparing and contrasting examples and concepts which look almost identical, but are not. It is also important that you ask yourself in what ways these things are similar and in what ways they differ. For example, the associative law of addition is similar in some respects to the associative law of multiplication, but different from it in others. Also, the two expressions $3 + 2 \cdot 4$ and $3 \cdot 2 + 4$ look similar, but are actually very different.

When you are working out exercises (or reading a concept), ask yourself, "What examples or concepts are similar to those which I am now doing? In what ways are they similar? How do I recognize the differences?" Doing this while you are working the exercises will help prevent you from making careless errors later on.

Coping with Getting Stuck

All of us have had the frustrating experience of getting stuck on a problem; sometimes even the simple problems can give us difficulty.

Perhaps you do not know how to begin; or you are stuck halfway through an exercise and are at a loss as to how to continue; or your answer and the book's answer do not seem to match. (Do not assume the book's solutions are 100% correct—we are only human even if we are math teachers. But do be sure to check that you have copied the problem accurately.)

Assuming you have reviewed all the relevant material beforehand, be sure you have spent enough time on the problem. Some people take one look at a problem and simply give up without giving the problem much thought. This is not what we regard as "getting stuck," since it is giving up before having even gotten started.

If you find after a reasonable amount of time, effort, and *thought*, that you are still not getting anywhere, and if you have looked back through your notes and textbook and still have no clue as to what to do, try to find exercises similar to the one you are stuck on (with answers in the back) that you can do. Analyze what you did to arrive at the solution and try to apply those principles to the problem you are finding difficult. If you have difficulty with those similar problems as well, you may have missed something in your notes or in the textbook. Reread the material and try again. If you are still not successful, go on to different problems or take a break and come back to it later.

If you are still stuck, wait until the next day. Sometimes a good night's rest is helpful. Finally, if you are still stuck after rereading the material, see your teacher (or tutor) as soon as possible.

Reviewing Old Material

One of the most difficult aspects of learning algebra is that each skill and concept depend on those previously learned. If you have not acquired a certain skill or learned a particular concept well enough, this will more than likely affect your ability to learn the next skill or concept.

Thus, even though you have finished a topic that was particularly difficult for you, you should not breathe too big a sigh of relief. Eventually you will have to learn that topic well in order to understand subsequent topics. It is important that you try to master all skills and understand all concepts.

Whether or not you have had difficulty with a topic, you should be constantly reviewing previous material as you continue to learn new subject matter. Reviewing helps to give you a perspective of the material you have covered. It helps you tie the different topics together and makes them *all* more meaningful. Some statement you read 3 weeks ago, and which may have seemed very abstract then, is suddenly simple and obvious in the light of all you now know.

It is also a good idea to distribute your study sessions over a period of time. That is, instead of putting in 6 hours in one day and none the next two days, put in 2 hours each day over the three days. You will find that not only will your studying be less boring, but also you will retain more with less effort.

As we mentioned before, your study activity should be varied during a study session. It is also a good idea to take short breaks and relax. A study "hour" could consist of 50 minutes of studying and a 10-minute break.

Study Activities

If you are going to learn algebra well enough to be able to demonstrate high levels of performance on exams, then you must concern yourself with both developing your skills in algebraic manipulation and understanding what you are doing and why you are doing it.

Many students concentrate only on skills and resort to memorizing the procedures for algebraic manipulations. This may work for quizzes or a test covering just a few topics. For exams covering a chapter's worth of material or more this can be quite a burden on the memory. Eventually interference occurs and problems and procedures get confused. If you find yourself doing well on quizzes but not on longer exams, this may be your problem.

Concentrating on understanding what a method is and why it works is important. Neither the teacher nor the textbook can cover every possible way in which a particular concept may present itself in a problem. If you understand the concept, you should be able to recognize it in a problem. But again, if you concentrate only on understanding concepts and not on developing skills, you may find yourself prone to making careless and costly errors under the pressure of an exam.

In order to achieve the goal of both skill development and understanding, your studying should include four activities: **(1)** practicing problems, **(2)** reviewing your notes and textbook, **(3)** drilling with study cards (to be discussed in the next section), and **(4)** reflecting on the material being reviewed and the exercises being done.

Rather than doing any one of these activities over a long period of time, it is best to do a little of the first three activities during a study session and save some time for reflection at the end of the session.

Making Study Cards

Study cards are 3″ × 5″ or 5″ × 8″ index cards which contain summary information needed for convenient review. We will discuss three types of cards: the definition principle card, the warning card, and the quiz card.

The ***definition/principle* (D/P) *cards*** contain a single definition, concept, or rule for a particular topic. The front of each D/P card should contain the following:

1. A heading of a few words
2. The definition, concept, or rule accurately recorded
3. If possible, a restatement of the definition, concept, or rule in your own words

The back of the card should contain examples illustrating the idea on the front of the card.

Here is an example of a D/P card.

FRONT

The Distributive Property
$a(b+c) = ab + ac$
or
$(b+c)a = ba + ca$
Multiply each term by a.

BACK

(1) $3x(x+2y) = 3x(x) + 3x(2y)$
$= 3x^2 + 6xy$
(2) $-2x(3x-y) = -2x(3x) - 2x(-y)$
$= -6x^2 + 2xy$
(3) $(2x+y)(x+y) = 2x(x+y) + y(x+y)$

Warning* (W) *cards contain errors that you may be consistently making on homework, quizzes, or exams, or those common errors pointed out by your teacher or your text. The front of the warning card should contain the word WARNING; the back of the card should contain an example of both the correct and incorrect way an example should be done. Be sure to label clearly which solution is correct and which is not.

For example:

FRONT

WARNING
EXPONENTS
An exponent refers only to the factor immediately to the left of the exponent.

BACK

EXAMPLES
$2 \cdot 3^2 = 2 \cdot 3 \cdot 3$ *NOT* $(2 \cdot 3)(2 \cdot 3)$
$(-3)^2 = (-3)(-3) = 9$
↑ Parentheses mean -3 is the factor to be squared.
BUT
$-3^2 = -3 \cdot 3 = -9$
↑ The factor being squared here is 3, not -3.

Pretend it is a real test; that is, do not leave your seat or look at your notes, books, or answers until your time is up. (Before giving yourself a test you may want to refer to next chapter's discussion on taking exams.)

When your time is up, stop; you may now look up the answers and grade yourself. If you are making errors, check over what you are doing wrong. Find the section where those problem types are covered, review the material, and try more problems of that type.

If you do not finish your practice test on time, you should definitely work on your speed. Remember that speed, as well as accuracy, is important on most exams.

Think about what you were doing as you took your test. You may want to change your test-taking strategy or reread the next chapter's discussion on taking exams. If you were not satisfied with your performance and you have the time after the review, make up and give yourself another practice test.

Taking an Algebra Exam

Just Before the Exam

You will need to concentrate and think clearly during the exam. For this reason it is important that you get plenty of rest the night before the exam, and that you have adequate nourishment.

It is *not* a good idea to study up until the last possible moment. You may find something that you missed and become anxious because there is not enough time to learn it. Then rather than simply missing a problem or two on the exam, the anxiety may affect your performance on the entire exam. It is better to stop studying some time before the exam and do something else. You could, however, review formulas you need to remember and warnings (common errors you want to avoid) just before the exam.

Also, be sure to give yourself plenty of time to get to the exam.

Beginning the Exam

At the exam, make sure that you listen carefully to the instructions given by your instructor or the proctor.

As soon as you are allowed to begin, jot down the formulas you think you might need, and write some key words (warnings) to remind you to avoid common errors or errors you have previously made. Writing down the formulas first will relieve you of the burden of worrying about whether you will remember them when you need to, thus allowing you to concentrate more.

You should refer back to the relevant warnings as you go through the exam to make sure you avoid those errors.

Remember to read the directions carefully.

What to Do First

Not all exams are arranged in ascending order of difficulty (from easiest to most difficult). Since time is usually an important factor, you do not want to spend so much time working on a few problems that you find difficult and then find that you do not have enough time to solve the problems that are easier for you. Therefore, it is strongly recommended that you first look over the exam and then follow the order given below:

1. Start with the problems which you know how to solve quickly.
2. Then go back and work on problems which you know how to solve but take longer.
3. Then work on those problems which you find more difficult, but for which you have a general idea of how to proceed.
4. Finally, divide the remaining time between the problems you find most difficult and checking your solutions. Do not forget to check the warnings you wrote down at the beginning of the exam.

You probably should not be spending a lot of time on any single problem. To determine the average amount of time you should be spending on a problem, divide the amount of time given for the exam by the number of problems on the exam. For example, if the exam lasts for 50 minutes and there are 20 problems, you should spend an average of $\frac{50}{20} = 2\frac{1}{2}$ minutes per problem. Remember, this is just an estimate. You should spend less time on "quick" problems (or those worth fewer points), and more time on the more difficult problems (or those worth more points). As you work the problems be aware of the time; if half the time is gone you should have completed about half of the exam.

Dealing with Panic

In the first two chapters of this text we have given you advice on how to learn algebra. In the last chapter we discussed how to prepare for an algebra exam. If you followed this advice and put the proper amount of time to good use you should feel fairly confident and less anxious about the exam. But you may still find during the course of the exam that you are suddenly stuck or you "draw a blank." This may lead you to panic and say irrational things like "I'm stuck. . . . I can't do this problem. . . . I can't do any of these problems. . . . I'm going to fail this test." Your heart may start to beat faster and your breath may quicken. You are entering a panic cycle.

These statements are irrational. Getting stuck on a few problems does not mean that you cannot do any algebra. These statements only serve to interfere with your concentrating on the exam itself. How can you think about solving a problem while you are telling yourself that you cannot? The increased heart and breath rate are part of this cycle.

9

The trouble with self-esteem

BY JOHN LEO

Most people think of the California state task force on self-esteem as yet another California joke, one more zany feel-good perpetration by lotus land's blissed-out mental surfers. This notion has been encouraged by Garry Trudeau, the *Doonesbury* cartoonist, who poked savage fun at the task force when it was announced and again when it issued its final report three years and $735,000 later.

Any report devoted to the idea that the state should go around promoting and monitoring good feelings is obviously open to ridicule. (The task force deflected some gibes and criticism by adding "personal and social responsibility" to its title and by issuing a few conservative findings.) But the conception of self-esteem as a public-policy issue is not a lotus-land joke, nor a California-only phenomenon. It is an idea that has quietly taken hold all around the country. The self-esteem movement, in fact, is a social force of some strength, particularly in the schools. Rita Kramer, a New York journalist and author, conducted interviews at 20 education schools around the country and was startled to find that self-esteem is the dominant educational theory almost everywhere she went. She thinks the rising emphasis on feelings comes at the expense of subject matter and therefore is a very ominous development. Her book in progress has the nonsubtle working title *The Dumbing Down of American Education.*

BONNIE TIMMONS FOR USN&WR

Those who push self-esteem in the schools point out that the public-school system is in disastrous shape, particularly in the cities. Teachers are expected to cope with the devastating results of poverty, racial discrimination, crime, drugs, broken homes and child abuse. Under these wartime conditions, the schools are de facto social agencies, presumably with nothing to lose and much to gain by building up the egos of their children.

Self-esteem programs use simple exercises frankly borrowed from the "You're much too hard on yourself" California therapies. In the curriculum at St. Clement Catholic School in Somerville, Mass., children take part in "affirmation exercises," saying nice things about themselves, such as "I am a good person; I am special." Sometimes they do this silently while imagining themselves atop a windswept mountain; sometimes aloud in front of the class while looking into a mirror. They keep journals of their accomplishments, are encouraged to support the good feelings of classmates (the proper response is "Thanks—I affirm you for being a good friend") and glance up many times a day at the symbol of the program, a "potential bottle," a foot-high jar filled with blue water that represents the untapped possibilities in all children.

Cheap points of light. The Bush era turns out to be a perfect time for self-esteem programs. They cost almost nothing. They offer the light of sunny California optimism at a time of great pessimism. They are simple—easily grasped, easily spread. And in public-school systems torn by competing pressure groups, they have no natural enemies. They have only one flaw: They are a terrible idea.

First of all, despite the firsthand reports of many teachers, there is almost no research evidence that these programs work. The book *The Social Importance of Self-Esteem*, which is basically all the research turned up by the California task force, says frankly, "One of the disappointing aspects of every chapter in this volume . . . is how low the associations between self-esteem and its consequences are in research to date." In fact, those correlations are as close to zero as you can get in the social sciences. This confirms the common-sense judgment that behavior is rarely changed by injections of positive thinking and psychic boosterism. Confidence boosting has a long and important tradition in the schools, but what evidence we have indicates that fear of failure and parental hovering have much more to do with academic success than good feelings about the self.

Second, the self-esteem movement is on a collision course with the growing movement to revive the schools academically. The self-esteem movement is rooted in the California therapies, which are sunny, feel-good and generally hostile to learning and intellect. Fritz Perls, the founder of Gestalt psychology, set the tone for California therapies by denouncing intellect as "a drag" and "a whore." The California task-force report is dedicated to the late Virginia Satir, a charismatic therapist with not much use for the human mind. ("She can fill any hall in the country, but she has great difficulty conceptualizing," one of her colleagues told me after a Satir lecture.)

The self-esteem literature is clotted with dismissive references to achievement. The self-esteem research book, mentioned above, contains many darts aimed at competition, achievement and success. After all, if people are perfect and lovable just the way they are, why should anyone need to change or strive?

This is why the obsession with self-esteem ultimately undermines real education. When the self-esteem movement takes over a school, teachers are under pressure to accept every child as is. To keep children feeling good about themselves, you must avoid all criticism and almost any challenge that could conceivably end in failure. In practice, this means each child is treated like a fragile therapy consumer in constant need of an ego boost. Difficult work is out of the question, and standards get lowered in school after school. Even tests become problematic because someone might fail them.

This becomes a parody of self-esteem. Real self-esteem is released when a child learns something and develops a sense of mastery. It is a byproduct of, and not a substitute for, real education. And until we grapple with the real agenda of the self-esteem movement—ersatz therapeutic massage instead of learning—there will probably be no educational reform at all. ■