ALGEBRA OF FUNCTIONS

DEFINITION:
The statement that f +g is the sum function of f and g means that $f+g = \{(x,y) / y = f(x) + g(x) , x \in D_f \cap D_g \}$.

DEFINITION:
The statement that f -g is the difference functions of f and g means $f-g = \{(x,y) / y = f(x) - g(x), x \in D_f \cap D_g \}$.

DEFINITION:
The statement that fg is the product function of f and g means $fg = \{(x,y) / y = f(x) g(x), x \in D_f \cap D_g \}$.

DEFINITION:
The statement that f/g is the quotient function of f and g means $f/g \{(x,y) / y = \frac{f(x)}{g(x)} , x \in D_g \cap D_g , g(x) \neq 0\}$.

DEFINITION:
The statement that f∘g is the composite function of g into f means $f \circ g = \{(x,y) / y = f[g(x)] , x \in D_g \text{ and } g(x) \in D_f \}$.

GRAPHING VARIATIONS OF BASIC FUNCTIONS

Given y = f(x) (Basic Function)
Assume $k > 0$

$y = f(x) + k$	Vertical shift up	
$y = f(x) - k$	Vertical shift down	
$y = f(x + k)$	Horizontal shift left	
$y = f(x - k)$	Horizontal shift right	
$y = -f(x)$	Reflection about horizontal	
$y = f(-x)$	Reflection about vertical	
$y = kf(x)$	$0 < \lvert k \rvert < 1$	Vertical shrink
	$\lvert k \rvert > 1$	Vertical stretch
$y = f(kx)$	$0 < \lvert k \rvert < 1$	Horizontal stretch
	$\lvert k \rvert > 1$	Horizontal shrink

Horizontal shrink = Vertical stretch

INVERSES OF FUNCTIONS AND RELATIONS

INVERSE

The statement that f^{-1} is the inverse of f means $(x,y) \in f^{-1}$ if and only if $(y,x) \in f$.

ONE-TO-ONE FUNCTION

The statement that f is a 1-to-1 function means if $(x_1,y_1) \in f$ and $(x_2,y_2) \in f$ and $x_1 \neq x_2$, then $y_1 \neq y_2$.

INVERSE FUNCTION

The statement that f^{-1} is the inverse <u>function</u> of f means 1) f is a one-to-one function, and 2) $(x,y) \in f^{-1}$ if and only if $(y,x) \in f$

EXAMPLES:

	Relation ?	Function?
f = {(3,4) , (-3,4) , (5,6) , (-5,6) , (2,3)}	Yes	Yes
g = {(3,4) , (4,5) , (5,6) , (6,7) , (-8,-7)}	Yes	Yes*
f^{-1} = {(4,3) , (4,-3), (6,5), (6,-5) , (3,2) }	Yes	No
g^{-1} = {(4,3) , (5,4) , (6,5) , (7,6) , (-7,-8)	Yes	Yes*

*one-to-one functions