MATH 1314

Tools to use when finding the zeros of a polynomial function:

1. Descartes' Rule of signs:

2. The rational zero test:

3. The boundedness theorem:

4. The complex conjugate zero theorem:

For each of the polynomial functions:

a. Use Descartes' rule of signs to find the number of positive and negative real zeros.

b. Use the rational zero test to determine the possible rational zeros of the function.

c. Find the rational zeros, if any (using the boundedness theorem will be beneficial if the rational zero list in (b.) is lengthy.

d. Find the other real zeros, if any and/or find the complex zeros , if any.

e. Find the x-intercepts of the graph, if any. (the real zeros in (c.) and (d.))

f. Find the y-intercept of the graph.

g. Graph the polynomial function using the leading coefficient test and using the multiplicity of the real zeros found in (c.) and (d.) to determine if the function crosses or touches the x-axis at each real zero.

1. $P(x) = x^4 + 3x^3 - 3x^2 - 11x - 6$

2. $P(x) = -2x^5 + 5x^4 + 34x^3 - 30x^2 - 84x + 45$

3. $P(x) = 2x^5 - 10x^4 + x^3 - 5x^2 - x + 5$

4. $P(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$

5. $P(x) = -2x^4 - x^3 + x + 2$

6. $P(x) = 4x^5 + 8x^4 + 9x^3 + 27x^2 + 27x$ (Hint: Factor out x first.)

7. $P(x) = 3x^4 - 14x^2 - 5$ (Hint: Factor the polynomial.)

8. $P(x) = -x^5 - x^4 + 10x^3 + 10x^2 - 9x - 9$

9. $P(x) = -3x^4 + 22x^3 - 55x^2 + 52x - 12$

ANSWERS

1. (a) positive zeros: 1; negative zeros: 3 or 1

   (b) $\pm 1, \pm 2, \pm 3, \pm 6$

   (c) -3, -1 (multiplicity 2), 2

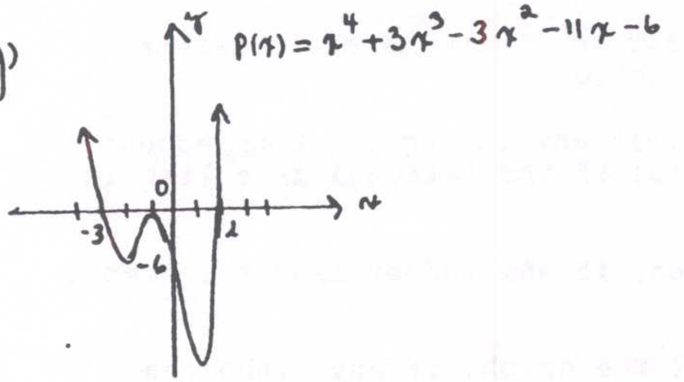
   (d) no other real zeros

   no other complex zeros

   (e) -3, -1, 2

   (f) -6

   (g)



2. (a) positive zeros: 3 or 1; negative zeros: 2 or 0

   (b) $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm 1/2, \pm 3/2, \pm 5/2, \pm 9/2, \pm 15/2, \pm 45/2$

   (c) -3, 1/2, 5

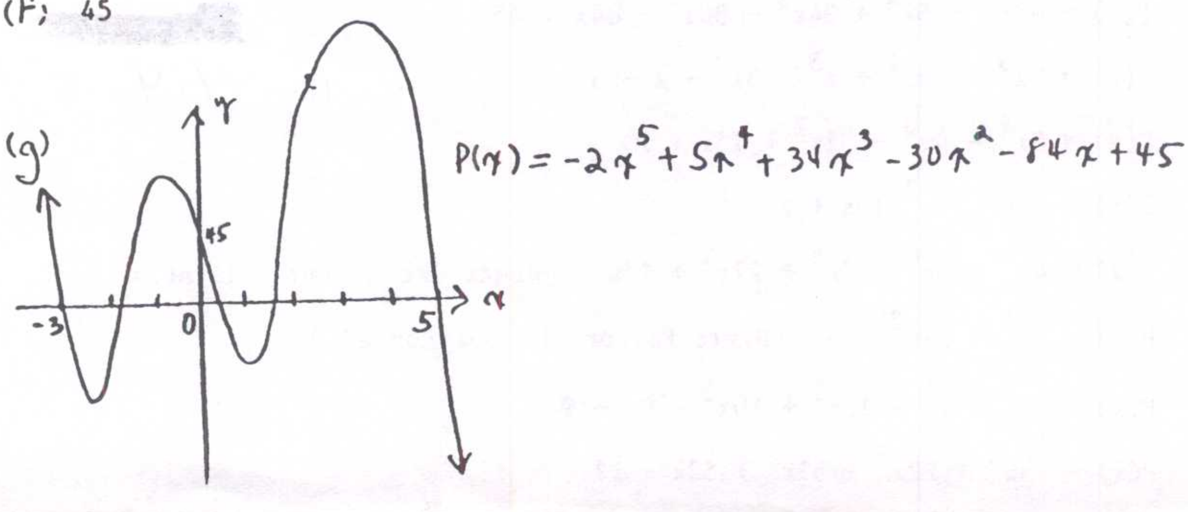
   (d) $-\sqrt{3}$, $\sqrt{3}$

   no other complex zeros

   (e) -3, 1/2, 5, $-\sqrt{3}$, $\sqrt{3}$

   (f) 45

   (g)



3. (a) positive zeros: 4, 2, or 0; negative zeros: 1

(b) ±1, ±5, ±1/2, ±5/2

(c) 5

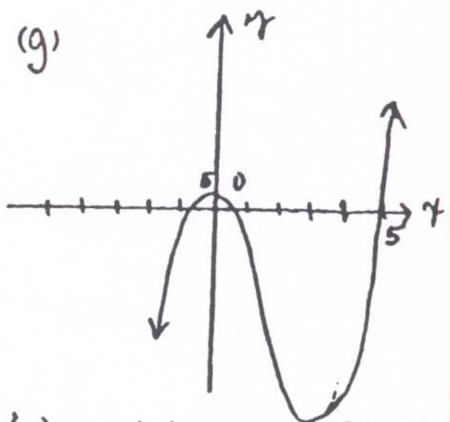
(d) $-\sqrt{2}/2$, $\sqrt{2}/2$

$-i$, i

(e) $-\sqrt{2}/2$, $\sqrt{2}/2$, 5

(f) 5

(g)



$P(x) = 2x^5 - 10x^4 + x^3 - 5x^2 - x + 5$

4. (a) positive zeros: 2 or 0; negative zeros: 2 or 0

(b) ±1, ±2, ±3, ±6, ±9, ±18, ±1/3, ±2/3, ±3/2

(c) $-2/3$, 3

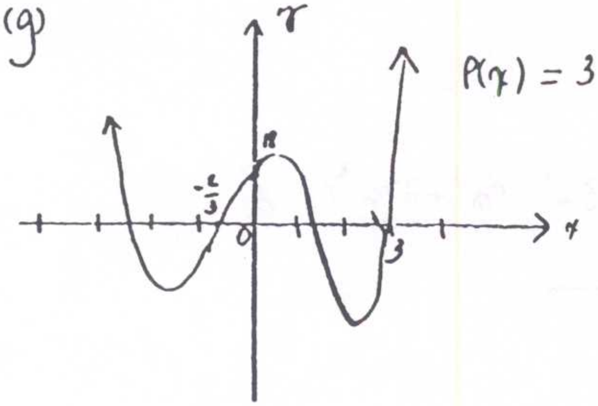
(d) $\frac{-1 + \sqrt{13}}{2}$, $\frac{-1 - \sqrt{13}}{2}$

no other complex zeros

(e) $-2/3$, 3, $\frac{-1 + \sqrt{13}}{2}$, $\frac{-1 - \sqrt{13}}{2}$

(f) 18

(g)



$P(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$

5. (a) positive zeros: 1; negative zeros: 3 or 1

(b) ±1, ±2, ±1/2

(c) -1, 1

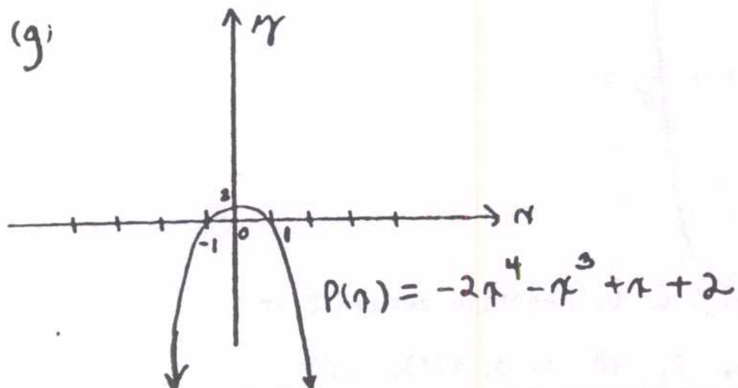
(d) no other real zeros

$-\frac{1}{4} + i\frac{\sqrt{15}}{4}, \quad -\frac{1}{4} - i\frac{\sqrt{15}}{4}$

(e) -1, 1

(f) 2

(g)



6. (a) positive zeros: 0; negative zeros: 4, 2, or 0

(b) 0, ±1, ±3, ±9, ±27, ±1/2, ±3/2, ±9/2, ±27/2, ±1/4, ±3/4, ±9/4, ±27/4

(c) 0, -3/2 (multiplicity 2)

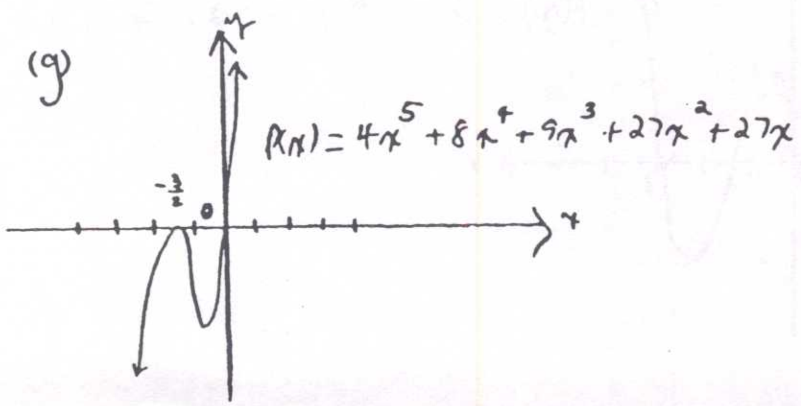
(d) no other real zeros

$\frac{1}{2} + i\frac{\sqrt{11}}{2}, \quad \frac{1}{2} - i\frac{\sqrt{11}}{2}$

(e) 0, -3/2

(f) 0

(g)



7. (a) positive zeros: 1; negative zeros: 1

(b) ±1, ±5, ±1/3, ±5/3

(c) no rational zeros

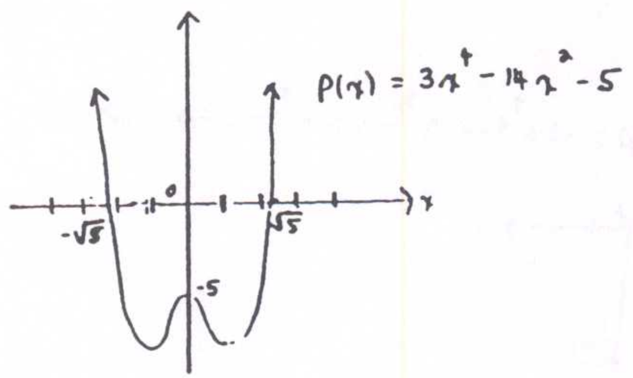
(d) $-\sqrt{5}$, $\sqrt{5}$

$-i\frac{\sqrt{3}}{3}$, $i\frac{\sqrt{3}}{3}$

(e) $-\sqrt{5}$, $\sqrt{5}$

(f) -5

(g)



8. (a) positive zeros: 2 or 0; negative zeros: 3 or 1

(b) ±1, ±3, ±9

(c) -3, -1(multiplicity 2), 1, 3

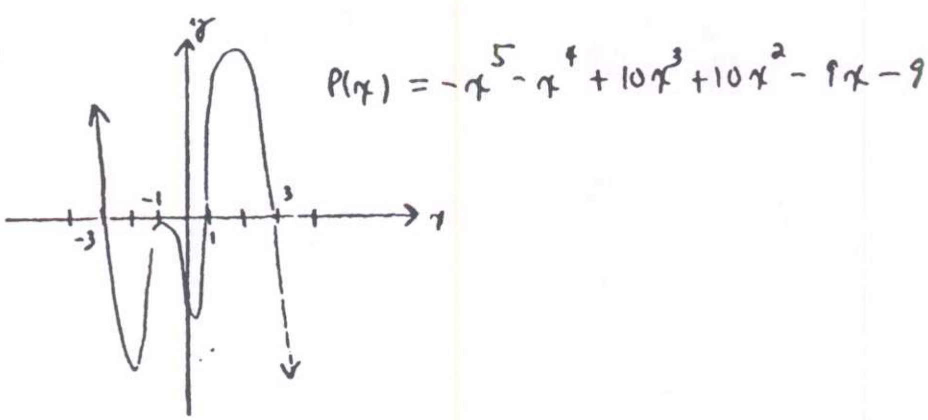
(d) no other real zeros

no other complex zeros

(e) -3, -1, 1, 3

(f) -9

(g)



9. (a) positive zeros: 4, 2, or 0; negative zeros: 0

   (b) ±1. ±2. ±3. ±4. ±6. ±12. ±1/3, ±2/3, ±4/3

   (c) 1/3, 2(multiplicity 2), 3

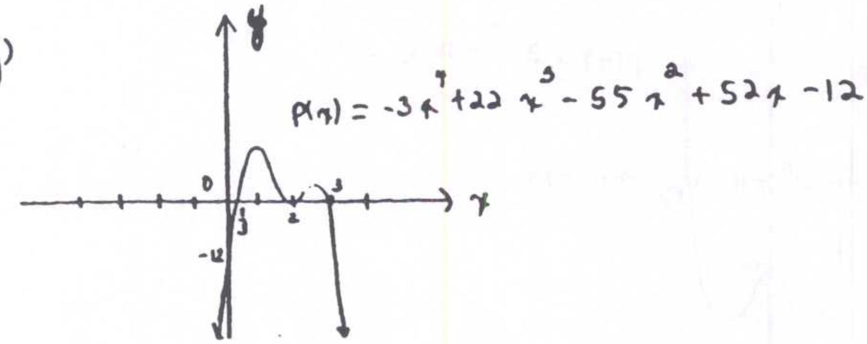
   (d) no other real zeros

   no other complex zeros

   (e) 1/3, 2, 3

   (f) -12

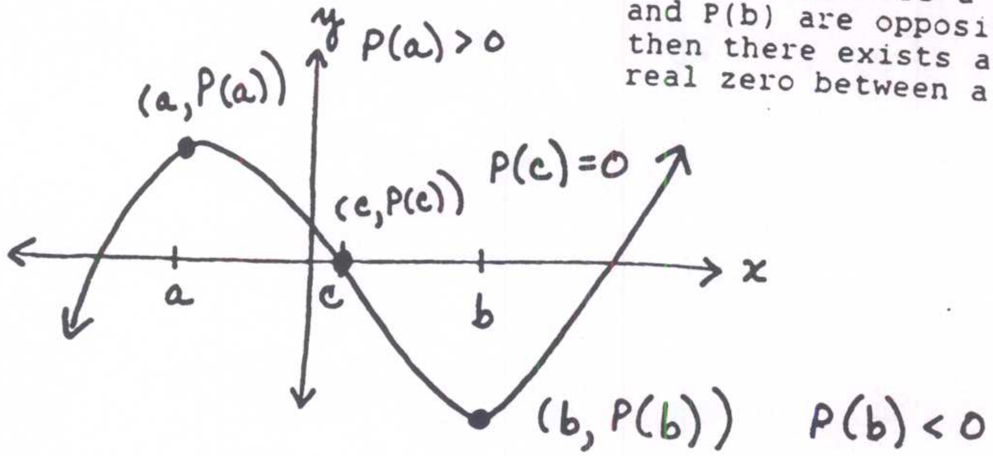
   (g)



Math 1314

The intermediate value theorem is useful in estimating zeros of polynomial functions that are irrational zeros.

__Intermediate__ __Value__ __Theorem:__ If $P(x)$ is a polynomial function with real coefficients, and if for real numbers a and b, $P(a)$ and $P(b)$ are opposite in sign, then there exists at least one real zero between a and b.



Use the intermediate value theorem to estimate the zero to the nearest tenth of each of the following polynomial functions within the given interval:

1. $P(x) = x^3 - 2x^2 - x + 1$; $[2, 3]$

2. $P(x) = x^4 - 6x^3 + 8x^2 + 2x - 1$; $[-1, 0]$

3. $P(x) = 8x^3 - 12x^2 + 2x + 1$; $[1, 2]$