College Algebra

Chapte 6 Section Two Examples

#2
$$\begin{cases} 2x - 4y + z = 3 \\ x - 3y + z = 5 \\ 3x - 7y + 2z = 12 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & -4 & 1 & 3 \\ 1 & -3 & 1 & 5 \\ 3 & -7 & 2 & 12 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 2 & -4 & 1 & 3 \\ 3 & -7 & 2 & 12 \end{array}\right] \quad R_1 \longleftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 0 & 2 & -1 & -7 \\ 0 & 2 & -1 & -3 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 0 & 1 & -\frac{1}{2} & -\frac{7}{2} \\ 0 & 2 & -1 & -3 \end{array}\right] \frac{1}{2}R_2 \to R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & -\frac{11}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{7}{2} \\ 0 & 0 & 0 & 4 \end{array}\right] \begin{array}{l} 3R_2 + R_1 \to R_1 \\ \\ -2R_2 + R_3 \to R_3 \end{array}$$

No Solution or $\emptyset$

#4 $\begin{cases} 5x - 11y + 6z = 12 \\ -x + 3y - 2z = -4 \\ 3x - 5y + 2z = 4 \end{cases}$

$$\left[\begin{array}{ccc|c} 5 & -11 & 6 & 12 \\ -1 & 3 & -2 & -4 \\ 3 & -5 & 2 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 5 & -11 & 6 & 12 \\ 1 & -3 & 2 & 4 \\ 3 & -5 & 2 & 4 \end{array}\right] -R_2 \to R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 5 & -11 & 6 & 12 \\ 3 & -5 & 2 & 4 \end{array}\right] R_1 \longleftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 4 & -4 & -8 \\ 0 & 4 & -4 & -8 \end{array}\right] \begin{array}{l} -5R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 1 & -1 & -2 \\ 0 & 4 & -4 & 8 \end{array}\right] \frac{1}{4}R_2 \to R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} 3R_2 + R_1 \to R_1 \\ -4R_2 + R_3 \to R_3 \end{array}$$

Infinitely Many Solutions

Let $z = t$ ⟹

$x - z = -2$
$y - z = -2$

$x - t = -2$
$y - t = -2$

$x = t - 2$
$y = t - 2$

Parametric Form

$\{(t-2, t-2, t)\}$

t is any real ✝