Math 1314
Chapter 8 Section 4
Mathematical Induction

Students please read pages 633 through 640. Pay close attention to page 635 and learn the Principle of Mathematical Induction (PMI) and the Steps in a Proof by Mathematical Induction.

Examples from Exercise 8.4

2. $S_n : 3+4+5+...+(n+2) = \frac{n(n+5)}{2}$

   $S_1 : 3 = \frac{1(1+5)}{2}$

   $S_2 : 3+4 = \frac{2(2+5)}{2}$

   $7 = 7$

   $S_3 : 3+4+5 = \frac{3(3+5)}{2}$

   $12 = 12$

4. $S_n : 3$ is a factor of $n^3 - n$

   $S_1 : 3$ is a factor of $1^3 - 1 = 0$

   $S_2 : 3$ is a factor of $2^3 - 2 = 6$

   $S_3 : 3$ is a factor of $3^3 - 3 = 24$

6. $S_n : 3+4+5+...+(n+2) = \frac{n(n+5)}{2}$

   $S_k : 3+4+5+...+(k+2) = \frac{k(k+5)}{2}$

   $S_{k+1} : 3+4+5+...+[(k+1)+2] = \frac{(k+1)[(k+1)+5]}{2}$

10. $S_n : 2$ is a factor of $n^2 - n$

$S_k : 2$ is a factor of $k^2 - k$

$S_{k+1} : 2$ is a factor of $(k+1)^2 - (k+1)$

12. $S_n : 3 + 4 + 5 + ... + (n+2) = \frac{n(n+5)}{2}$

Proof: a. For n = 1

$$S_1 : 3 = \frac{1(1+5)}{2} = 3$$

Therefore S_1 is true.

b. Assume S_k is true, then $3 + 4 + 5 + ... + (k+2) = \frac{k(k+5)}{2}$

c. Consider $S_{k+1} : 3 + 4 + 5 + ... + [(k+1) + 20] = \frac{(k+1)[(k+1)+5]}{2}$

$$3 + 4 + 5 + ... + (k+2) + [(k+1) + 2]$$
$$= \frac{k(k+5)}{2} + (k+3) = \frac{k(k+5) + 2(k+3)}{2}$$
$$= \frac{k^2 + 5k + 5k + 6}{2} = \frac{k^2 + 7k + 6}{2}$$
$$= \frac{(k+1)(k+6)}{2} = \frac{(k+1)[(k+1)+5]}{2}$$

Since S_1 is true and $S_k \to S_{k+1}$, then S_n is true for all positive integers n.

18. $1 + 3 + 3^2 + ... + 3^{n-1} = \frac{3^n - 1}{2}$

Proof: a. $S_1 : 1 = \frac{3^1 - 1}{2} = 1$

Therefore S_1 is true.

b. Assume S_k is true, then $1 + 3 + 3^2 + ... + 3^{k-1} = \frac{3^k - 1}{2}$

c. Consider $S_{k+1}: 1+3+3^2+\ldots 3^{(k+1)-1} = \dfrac{3^{k+1}-1}{2}$

$$1+3+3^2+\ldots 3^{k-1}+3^k = \frac{3^k-1}{2}+3^k$$
$$= \frac{3^k-1}{2}+\frac{2(3^k)}{2} = \frac{3(3^k)-1}{2}$$
$$= \frac{3^{k+1}-1}{2}$$

Since S_1 is true and $S_k \to S_{k+1}$, then S_n is true for all positive integers n.

26. $S_n: 2$ is a factor of n^2+3n

Proof: a. $S_1: 2$ is a factor of $1^2+3(1)=4$

S_1 is true.

b. Assume S_k is true, then 2 is a factor of k^2+3k

c. $S_{k+1}: 2$ is a factor of $(k+1)^2+3(k+1)$

$$S_{k+1}: (k+1)^2+3(k+1) = k^2+2k+1+3k+3$$
$$= (k^2+3k)+2k+4 = 2q+2k+4 \qquad \text{q, k, and 2 are integers}$$
$$= 2(q+k+2)$$

Therefore S_1 is true and $S_k \to S_{k+1}$, then S_n is true for all positive integers n.

Assignment: Ex 8.4 odd 1 -37

Submit a proof of the following for 20 points. The score will replace your lowest lab grade.

$$S_n: 1^3+2^3+3^3+\ldots+n^3 = \frac{n^2(n+1)^2}{4}$$