APPLICATION 7.3: Archimedes Principle

See Section 7.2
Volume: The Disc Method
Calculus, 4th Edition,
Larson/Hostetler/Edwards

The Archimedes Principle states that the upward or buoyant force on an object within a fluid is equal to the weight of the fluid that the object displaced. Recall that weight is the special name given to the gravitational force an object experiences near the surface of the earth. There are two cases to consider when calculating this force. One case is when the object is entirely submerged. The other, more interesting case, is when the object is floating, that is, only partially submerged.

Consider the case of the partially submerged object. We can obtain information about the relative densities of the floating object and the fluid by observing how much of the object is above the surface of the fluid and how much is below the surface of the fluid. We can also determine how large a floating object is if we know the amount that is above the surface and we know the relative densities.

In the first reported practical use the Archimedes Principle, Archimedes was asked to determine whether a crown was real gold or a fake. This is not a trivial problem because the crown was not a simple shape. Archimedes thought for a long time before he came up with his ingenious solution. He weighed the crown in and out of water and from that, was able to conclude that the crown was fake!

Tip of the Iceberg

Suppose we can see the top of a floating iceberg. We know the density of ice is 0.92×10^3 kg/m^3 and of ocean water is 1.03×10^3 kg/m^3. Using Archimedes Principle, what is the total size of the iceberg?

SOLUTION

Consider some object of uniform density floating in a fluid. (See Figure 7.4.)

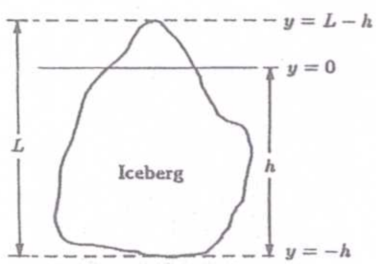

FIGURE 7.4

Let ρ_f be the density of the fluid and let ρ_o be the density of the object. The buoyant force is given by

$$F = \rho_f g \int_{-h}^{0} A(y)\, dy$$

where $A(y)$ is a typical cross section and g is the acceleration due to gravity. The weight of the object is

$$W = \rho_o g \int_{-h}^{L-h} A(y)\, dy.$$

We also know by the Archimedes Principle that when the object is floating $F = W$, so

$$\rho_f g \int_{-h}^{0} A(y)\, dy = \rho_o g \int_{-h}^{L-h} A(y)\, dy.$$

This gives the ratio of densities

$$\frac{\rho_o}{\rho_f} = \frac{\text{submerged volume}}{\text{total volume}}.$$

Since, for ice, the density is 0.92×10^3 kilograms/cubic meter and for ocean water, the density is 1.03×10^3 kilograms/cubic meter, about 89% of the iceberg is below the surface.