

Orthogonal Trajectory

Given family of curves (parabolas)

$$y^2 = 2Cx \quad C = \frac{y^2}{2x}$$

$$2yy' = 2C$$

$$y' = \frac{C}{y} = \frac{y^2}{2x} \cdot \frac{1}{y} = \frac{y}{2x}$$

Orthogonal Trajectories (ellipse)

$$y' = -\frac{2x}{y}$$

$$yy' = -2x$$

$$\int y dy = -2 \int x dx$$

$$\frac{y^2}{2} = -x^2 + K_1$$

$$2x^2 + y^2 = K$$