Optimization

#56.

$V = \frac{1}{3}\pi r^2 h$

$h = \sqrt{144 - r^2}$

$V = \frac{1}{3}\pi r^2 \sqrt{144 - r^2}$

$\frac{dV}{dr} = \frac{1}{3}\pi (2r)\sqrt{144 - r^2} + \frac{1}{3}\pi r^2 \left(\frac{1}{2}\right)(144 - r^2)^{-\frac{1}{2}}(-2r) = 0$

$\frac{1}{3}\pi \left[\frac{288r - 2r^3 - r^3}{\sqrt{144 - r^2}} \right] = 0$

$288r - 3r^3 = 0$

$96r - r^3 = 0$

$r(96 - r^2) = 0$

$r = 0 \quad r = \sqrt{96} = 4\sqrt{6}$

$h = \sqrt{144 - 96} = \sqrt{48} = 4\sqrt{3}$

V is maximum when $r = 4\sqrt{6}$ & $h = 4\sqrt{3}$

Area of circle radius 12 is 144π.

Lateral Surface area of a cone is $\pi(4\sqrt{6})\sqrt{(4\sqrt{6})^2 + (4\sqrt{3})^2}$

$= (48\sqrt{6})\pi$

Area of a sector $= 144 - 48\sqrt{3} = \frac{1}{2}\theta r^2$

$= \frac{1}{2}(12)^2\theta = 72\theta$

$$\theta = \frac{144\pi - 48\sqrt{3}\pi}{72} = \frac{2\pi}{3}(3 - \sqrt{3})$$

≈ 1.153 radians

$= 66°$