MATH 2415
CALCULUS III
CHAPTER FOURTEEN POSTTEST SUMMER 2004

SHOW ALL OF YOUR WORK AND CIRCLE YOUR ANSWERS.

1. Find the divergence of the vector field

$\mathbf{F}(x, y, z) = \cos(xy)\mathbf{i} - \sin(xz)\mathbf{k}$ at the point $\left(1, \frac{\pi}{2}, 0\right)$.

a. $-\frac{\pi}{2}$

b. $\frac{\pi}{2} - 1$

c. None of these

d. $-\frac{\pi}{2} + 1$

e. -1

2. Find the curl of $\mathbf{F}(x, y, z) = xy\mathbf{i} - z\mathbf{j} + x\mathbf{k}$ at the point (1, 0, 1).

a. None of these

b. $\mathbf{j} - \mathbf{k}$

c. $\mathbf{j} + \mathbf{k}$

d. $\mathbf{i} - \mathbf{j} - \mathbf{k}$

e. $-\mathbf{j} - \mathbf{k}$

3. Evaluate $\int_C (x^3y + 1)\, ds$ if C is the path counterclockwise around the upper half of the unit circle.

a. $-\frac{1}{2} + \pi$

b. $\frac{1}{2} + \pi$

c. None of these

d. $-\frac{1}{4} + \pi$

e. $\pi - 1$

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4. Evaluate $\int_C xy\,dx - 5x^2y\,dy$ where the curve C is represented by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$, for $0 \le t \le 1$.

a. $-\dfrac{17}{12}$
b. None of these
c. -2
d. $-\dfrac{3}{4}$
e. -4

5. For the force field $\mathbf{F}(x, y, z) = (y\cos xy + yz)\mathbf{i} + (x\cos xy + xz)\mathbf{j} + xy\mathbf{k}$, calculate the work done by $\mathbf{F}$ on an object moving along a curve from the point $\left(1, \frac{\pi}{2}, 2\right)$ to the point $\left(\pi, \frac{1}{2}, 1\right)$.

a. $\dfrac{\pi}{2}$
b. $1 - \pi$
c. $1 - \dfrac{\pi}{2}$
d. $-\pi$
e. None of these

6. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = xy\mathbf{j}$ and C is the semi-circle $x^2 + y^2 = 9$, $y \ge 0$, and the line $y = 0$, $-3 \le x \le 3$.

a. 18
b. 6
c. 12
d. None of these
e. 24

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7. Find a vector-valued function whose graph is the elliptic paraboloid given by $z = \frac{x^2}{4} + \frac{y^2}{9}$.

a. $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + \left[\frac{u^2}{4} + \frac{v^2}{9}\right]\mathbf{k}$

b. $\mathbf{r}(u, v) = v\mathbf{i} + u\mathbf{j} + \left[\frac{u^2}{4} + \frac{v^2}{9}\right]\mathbf{k}$

c. None of these

d. $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + uv\mathbf{k}$

e. $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + (9u^2 + 4v^2)\mathbf{k}$

8. Evaluate the surface integral $\iint_S x\, dS$ if S is the part of the plane $z = 4 - 2x - 2y$ in the first octant.

a. 9
b. None of these
c. 8
d. 6
e. 4

9. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$ where $\mathbf{F}(x, y, z) = x\mathbf{i} + 2y\mathbf{j} + e^z\mathbf{k}$ and S is the surface of the cube $\{(x, y, z) : 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\}$.

a. e
b. $e^3 + 3$
c. $e + 3$
d. None of these
e. $e + 2$

10. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = 2z\mathbf{i} - x\mathbf{j} + 3y\mathbf{k}$, and S is the portion of the plane (oriented upward) $3x + 3y + 2z = 6$ in the first octant and C is its boundary.

a. 16
b. 14
c. None of these
d. 13
e. 18