

## The Chain Rule

$$f(x) = (3x+2)^2$$

$$f'(x) = 2(3x+2)(3)$$

$$= 6(3x+2) = 18x+12$$

$$\begin{aligned}
 & \int (18x + 12) dx && \text{Let } u = 3x + 2 \\
 & = 6 \int (3x + 2) dx && du = 3 dx \\
 & = 6 \left( \frac{1}{3} \right) \int (3x + 2) 3 dx \\
 & = 2 \int u du = 2 \frac{u^2}{2} + C \\
 & = u^2 + C = \underline{(3x + 2)^2 + C}
 \end{aligned}$$

$$\int \sin(2x) dx$$

$$\text{Let } u = 2x$$

$$du = 2 dx$$

$$= \frac{1}{2} \int [\sin(2x)] (2 dx)$$

$$= \frac{1}{2} \int \sin u du = -\frac{1}{2} \cos u + C$$

$$= -\frac{1}{2} \cos 2x + C$$

$$\#16 \int t^3 \sqrt{t^4 + 5} dt$$

$$= \int t^3 (t^4 + 5)^{\frac{1}{2}} dt$$

$$\begin{aligned} \text{Let } u &= t^4 + 5 \\ du &= 4t^3 dt \end{aligned}$$

$$= \frac{1}{4} \int (t^4 + 5)^{\frac{1}{2}} (4t^3) dt$$

$$= \frac{1}{4} \int u^{\frac{1}{2}} du = \frac{1}{4} \left( \frac{2}{3} u^{\frac{3}{2}} \right) + C$$

$$= \frac{1}{6} (t^4 + 5)^{\frac{3}{2}} + C$$

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$$\begin{aligned} \text{Let } u &= \cos x \\ du &= -\sin x dx \end{aligned}$$

$$\int \frac{\sin x}{\cos^2 x} dx = \int \cos^{-2} x \cdot \sin x dx$$

$$= \int u^{-2} du = -\frac{1}{2} u^{-1} + C$$

$$= \frac{-1}{2u} + C = \frac{-1}{2\cos x} + C$$

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$$\int_0^2 x \sqrt[3]{4+x^2} dx$$

$$= \int_0^2 (4+x^2)^{\frac{1}{3}} dx$$

$$= \frac{1}{2} \int_0^2 (4+x^2)^{\frac{1}{3}} (2x) dx$$

$$= \frac{1}{2} \int_4^8 u^{\frac{1}{3}} du = \frac{1}{2} \cdot \frac{3}{4} u^{\frac{4}{3}} \Big|_4^8$$

$$= \frac{3}{8} [8^{\frac{4}{3}} - 4^{\frac{4}{3}}] = \frac{3}{8} [16 - (\sqrt[3]{4})^4] \approx \textcircled{3.62}$$

$$2x = u = 4 + x^2$$

$$du = 2x dx$$

$$u(0) = 4$$

$$u(2) = 8$$

$$f(x) = (x-2)^2 \quad (2, 5)$$

$$\frac{5-2}{2(4)} = \frac{3}{8}$$

$$\Delta x = \frac{5-2}{4} \\ = \frac{3}{4}$$

$$\int_2^5 (x-2)^2 dx = \frac{3}{8} \left[ 0^2 + 2\left(\frac{3}{4}\right)^2 + 2\left(\frac{3}{2}\right)^2 + 2\left(\frac{9}{4}\right)^2 + (3)^2 \right]$$

$$\approx \textcircled{9.3}$$

$$\frac{5-2}{12} = \frac{3}{12} = \frac{1}{4}$$

$$\int_2^5 (x-2)^2 dx = \frac{1}{4} \left[ 0^2 + 4\left(\frac{3}{4}\right)^2 + 2\left(\frac{2}{2}\right)^2 + 4\left(\frac{9}{4}\right)^2 + (3)^2 \right]$$

$$= \textcircled{9}$$