Limit of a Cubic Function

Let $\epsilon > 0$ + show that $\lim_{x \to 2} x^3 + 5 = 13$ $\quad c = 2$, $L = 13$

Consider $0 < |x - c| < \delta$, then $0 < |x - 2| < \delta$

$|f(x) - L| < \epsilon$, then $|x^3 + 5 - 13| < \epsilon$

Observe: $|x^3 + 5 - 13| = |x^3 - 8| = |x^2 + 2x + 4|\,|x - 2|$

Let $\delta_1 = 1$ and choose $0 < |x - 2| < 1$

Then $-1 < x - 2 < 1$

$1 < x < 3 \qquad |x| < 3$

which implies $|x^2 + 2x + 4| < |3^2 + 2(3) + 4| = 19$

Choose $\delta_2 = \frac{\epsilon}{19}$ and choose $\delta_\epsilon = \min\{\delta_1, \delta_2\}$.

Choose x so that $0 < |x - 2| < \delta_\epsilon$ then

$0 < |x - 2| < \delta_1$ and $0 < |x - 2| < \delta_2$.

Finally $|x^3 + 5 - 13| = |x^3 - 8| = |x^2 + 2x + 4|\,|x - 2|$

$$< 19|x - 2| < \cancel{19}\left(\frac{\epsilon}{\cancel{19}}\right) = \epsilon$$

Therefore $\lim_{x \to 2} x^3 + 5 = 13$ by definition