Math 1325
Review Chapter 11 Sect. 10.5

For problems 1 through 3, graph f(x) by finding the following:

a. y intercept
b. x intercept(s)
c. vertical asymptotes
d. horizontal asymptotes
e. relative extrema

1. $f(x) = \dfrac{x^2 - 3x + 6}{x - 2}$

2. $f(x) = \dfrac{3}{(1 - x)^2}$

3. $f(x) = \dfrac{(x - 2)^2}{x^2}$

4. A recently release film has its weekly revenue given by $R(t) = \dfrac{50t}{t^2 + 36}$ where $t \geq 0$ and R(t) is in millions of dollars and t is in weeks.
   a. Graph R(t)
   b. When will revenue be maximized?
   c. Suppose that if revenue decreases for 4 consecutive weeks, the film will be removed from theaters and will be released as a video 12 weeks later. When will the video come out?

For problems 5 through 13, find y'.

5. $y = \ln(5\text{-}x)^4$

6. $y = \ln\sqrt{x^2 + 2}$

7. $y = \ln\left[(3x + 1)^5(x^3 - x)^4\right]$

8. $y = \ln\left[\dfrac{(x + 1)^2}{x^3}\right]$

9. $y = \ln\left[\dfrac{x - 1}{x + 1}\right]^3$

10. $y = x^2 \ln(2x + 1)$

11. $y = \log(x^3 - 5)$

12. $y = \log_3\left(\dfrac{x^2 + 1}{x^2}\right)$

13. $y = \sqrt{\ln(3x + 1)}$

14. The total revenue function is $R(x) = \dfrac{\ln x^2}{x}$

Find the marginal revenue function and the number of units sold to have the maximum revenue.

15. The supply function for a certain product is given by $q = 10 + 5\ln(p^2+1)$, where q= the numbers of units supplied at price p. Find the rate of change of supply when the price is \$5, and interpret your answer.

In problems 16 through 25, find $\dfrac{dy}{dx}$.

16. $y = \sqrt{3 + e^{2x}}$

17. $y = \dfrac{2}{2x + e^{2x}}$

18. $y=(e^x+3e^{4x})^3$

19. $y = 4e^{x^2}$

20. $y = 5^{x^2-1}$

21. $y = \dfrac{e^{4x}}{2x + 1}$

22. $y = xe^{3x}$

23. $y = (3 + e^{4x})\left(2 - \dfrac{3}{e^x}\right)$

24. $y = (e^{2x} + 5)\ln(3x - 5)$

25. $y = \dfrac{1 + e^{2x}}{e^{3x}}$

26. In a monopoly market, the demand p for x units of an item is $p = 100e^{-.06x}$, Find the marginal revenue when x = 8 units.

In problem 27 through 29 find $\dfrac{dy}{dx}$

27. $x^3 - 2x^2y + 3xy^2 = 38$

28. $xe^y + 1 = xy$

29. $\ln xy = x + y$

30. Write the equation of the line tangent to the curve $3x^2 + 6xy - y^2 + 9 = 0$ at $(0,3)$

31. Find the points where $x^2 + 4y^2 - 10x - 11 = 0$ has horizontal and vertical tangents.

32. The demand function for a certain product is $p^2(q+1) = 5000$, where q is the quantity demanded at price p. Find the rate of change of price with respect to quantity demanded when 24.5 units are demanded. Interpret your result.

33. Assume x and y are differentiable functions of t for $x^2+xy+2=0$. Find $\frac{dy}{dt}$ if $\frac{dx}{dt} = -1$ when $x = 2$ and $y = -3$.

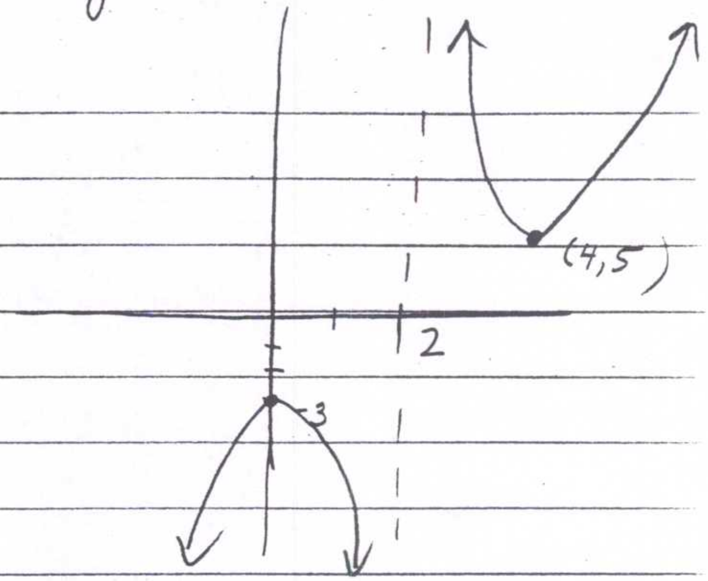
34. Suppose the cost function is given by $C(x) = 5000 + 2x$ and the revenue function is $R(x) = 10x - \frac{x^2}{1000}$ where x is the number of products produced in one week. If production is increasing at the rate of 500 products per week when production is 2000 products, find the rate of increase in profit.

35. If the demand function for a certain product is $p = -5q + 450$ and the supply function is $p = 2q + 170$, find the tax per item that will maximize the total tax revenue and find the maximum tax revenue.

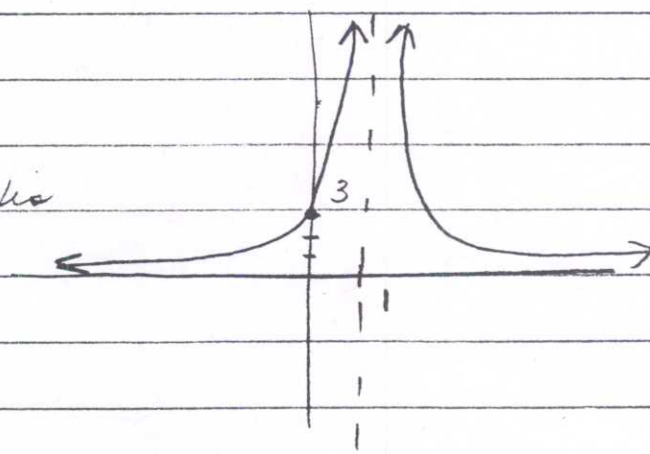
36. If the demand for a product is given by $p^2q = 27$, find the elasticity when the price is \$4. What type of elasticity is involved and how will a price increase affect total revenue?

Answer Key

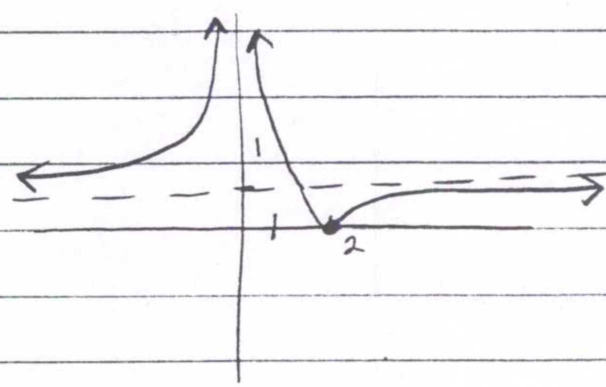
1. yint: (0, −3)
   xint: none
   vertical asymptate: $x=2$
   no horizontal asymptate
   relative max (0, −3)
   relative min (4, 5)



2. yint: (0, 3)
   xint: none
   vertical asymptates: $x=1$
   horizontal asymptate: x axis
   relative max none
   relative min none

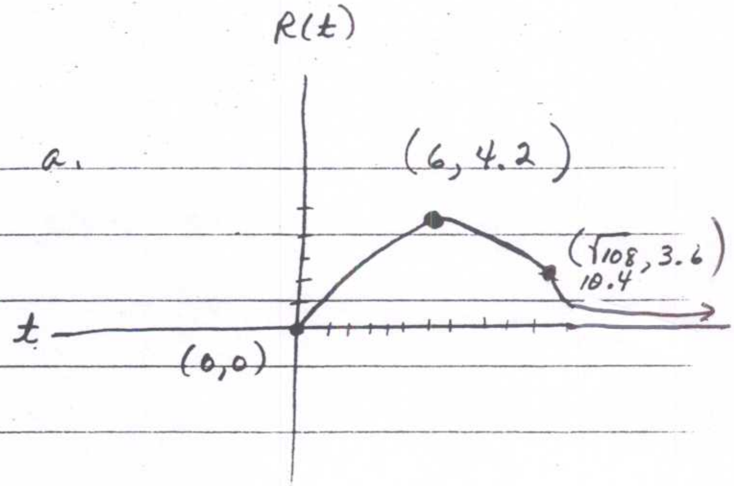


3. yint: none
   xint: (2, 0)
   vertical asymptate: $x=0$ (y axis)
   horizontal asymptate: $y=1$
   relative min: (2, 0)
   no relative max



4. yint (0, 0)
Xint (0, 0)
no vertical asymptote
horizontal asymptote: x axis
relative max $\left(6, \frac{25}{6}\right)$
inflection pt $(\sqrt{108}, 3.6)$

a.



b. the revenue will be maximized at the 6^{th} week

c. the revenue will begin decreasing after the 6th week, so the video will come out the 22nd week.

5. $y' = \frac{1}{(5-x)^4} \cdot 4(5-x)^3(-1) = \frac{-4}{5-x}$

6. $y' = \frac{1}{\sqrt{x^2+2}} \cdot \frac{1}{2}(x^2+2)^{-\frac{1}{2}}(2x) = \frac{x}{x^2+2}$

7. $y = \ln(3x+1)^5 + \ln(x^3-x)^4$

$y' = \frac{1}{(3x+1)^5} \cdot 5(3x+1)^4(3) + \frac{1}{(x^3-x)^4} \cdot 4(x^3-x)^3(3x^2-1)$

$y' = \frac{15}{3x+1} + \frac{4(3x^2-1)}{x^3-x}$

8. $y = \ln(x+1)^2 - \ln x^3$

$y' = \frac{1}{(x+1)^2} \cdot 2(x+1) - \frac{1}{x^3} \cdot 3x^2 = \frac{2}{x+1} - \frac{3}{x}$

9. $y = 3\ln\left[\frac{x-1}{x+1}\right] = 3\left[\ln(x-1) - \ln(x+1)\right]$

$y' = 3\cdot\left[\frac{1}{x-1} - \frac{1}{x+1}\right] = 3\cdot\frac{2}{(x-1)(x+1)} = \frac{6}{(x-1)(x+1)}$

10. $y' = x^2\cdot\frac{1}{2x+1}(2) + \ln(2x+1)(2x) = \frac{2x^2}{2x+1} + 2x\ln(2x+1)$

11. $y' = \frac{1}{x^3-5}\cdot 3x^2\cdot\frac{1}{\ln 10} = \frac{3x^2}{(\ln 10)(x^3-5)}$

12. $y' = \frac{1}{\ln 3}\left[\frac{1}{x^2+1}\cdot 2x - \frac{1}{x^2}\cdot 2x\right] = \frac{1}{\ln 3}\left[\frac{2x}{x^2+1} - \frac{2}{x}\right]$

13. $y' = \frac{1}{2}(\ln(3x+1))^{-1/2}\cdot\frac{1}{3x+1}\cdot 3 = \frac{3}{2(3x+1)\sqrt{\ln(3x+1)}}$

14. $R'(x) = \frac{2-\ln x^2}{x^2} = 0$ when $\ln x^2 = 2 \rightarrow 2\ln x = 2$

$\rightarrow \ln x = 1$

$\rightarrow x = e^1 \doteq 2.7$

Marginal revenue is $\frac{2-\ln x^2}{x^2}$

$\doteq 2.7$ unit

15. $q = 10 + 5\ln(p^2+1)$

$\frac{dq}{dp} = 5 \cdot \frac{1}{p^2+1} \cdot 2p = \frac{10p}{p^2+1}$. When $p = \$5$, $\frac{dq}{dp} = \frac{10(5)}{26} \doteq 1.92$

By increasing the price from \$5 to \$6, the quantity supplied will increase by 1.92 units

16. $y' = \frac{1}{2}(3+e^{2x})^{-1/2} \cdot e^{2x}(2) = \frac{e^{2x}}{\sqrt{3+e^{2x}}}$

17. $y = 2(2x+e^{2x})^{-1}$

$y' = 2(-1)(2x+e^{2x})^{-2}(2+e^{2x}(2)) = \frac{-2(2+2e^{2x})}{(2x+e^{2x})^2}$

18. $y' = 3(e^x + 3e^{4x})^2(e^x + 3e^{4x}(4)) = 3(e^x+3e^{4x})^2(e^x+12e^{4x})$

19. $y' = 4e^{x^2} \cdot 2x = 8xe^{x^2}$

20. $y' = 5^{x^2-1} \cdot (2x)\ln 5$

21. $y' = \frac{(2x+1)e^{4x}(4) - e^{4x}(2)}{(2x+1)^2}$

22. $y' = x \cdot e^{3x}(3) + e^{3x}(1)$

23. $y = (3+e^{4x})(2-3e^{-x})$

$y' = (3+e^{4x})(-3(-1)e^{-x}) + (2-3e^{-x})(e^{4x})(4)$

24. $y' = (e^{2x}+5) \cdot \frac{1}{3x-5} \cdot 3 + \ln(3x-5)\,(e^{2x}(2))$

25. $\frac{e^{3x}(e^{2x}(2)) - (1+e^{2x})e^{3x} \cdot 3}{e^{6x}} = y'$

26. $MR(8) = \$32.18$

27. $\frac{dy}{dx} = \frac{-3x^2+4xy-3y^2}{-2x^2+6xy}$

28. $\frac{dy}{dx} = \frac{y-e^y}{xe^y-x}$

29. $\frac{dy}{dx} = \frac{yx-y}{x-xy}$

34. $\frac{dP}{dt} = \$2000$ per week (Find $\frac{dP}{dt}$ if $\frac{dx}{dt} = 500$ and $x=2000$)

35. A tax of \$140 per item gives the maximum tax revenue of \$2800

36. $\eta = 2$; elastic, decrease

30. $6x + 6\left[x \cdot \frac{dy}{dx} + y(1)\right] - 2y\frac{dy}{dx} = 0$

$$\frac{dy}{dx}\left[6x - 2y\right] = -6x - 6y$$

$$\frac{dy}{dx} = \frac{-6x-6y}{6x-2y} \qquad \left.\frac{dy}{dx}\right|_{\substack{x=0\\y=3}} = \frac{-6(0)-6(3)}{6(0)-2(3)} = \frac{-18}{-6} = 3 = m$$

$$y - 3 = 3(x-0) \qquad y = 3x+3$$

31. $2x + 8y\frac{dy}{dx} - 10 = 0$

$$\frac{dy}{dx} = \frac{10-2x}{8y}$$

HORIZONTAL TANGENT $\rightarrow 10-2x=0$
$x=5$

VERTICAL TANGENT $\rightarrow 8y=0 \rightarrow y=0$

Horizontal tangent at $(5,3)$ & $(5,-3)$
vertical tangent at $(11,0)$ & $(-1,0)$

32. Find $\frac{dp}{dq}$. $\quad p^2(1) + (q+1)2p\cdot\frac{dp}{dq} = 0$

$$\frac{dp}{dq} = \frac{-p^2}{2p(q+1)} = -\frac{p}{2(q+1)}$$

$$\left.\frac{dp}{dq}\right|_{\substack{p=14\\q=24.5}} = \frac{-14}{2(25.5)} = -.27$$

33. $2x\frac{dx}{dt} + x\cdot\frac{dy}{dt} + y\cdot\frac{dx}{dt} = 0$

$$2(2)(-1) + 2\cdot\frac{dy}{dt} + -3(-1) = 0$$

$$-4 + 2\frac{dy}{dt} + 3 = 0 \qquad \frac{dy}{dt} = \frac{1}{2}$$