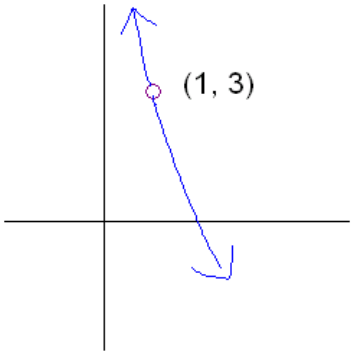
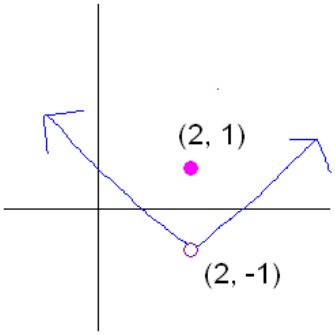
Math 1325
Chapter 9 Review
Section 1 through 6

I.

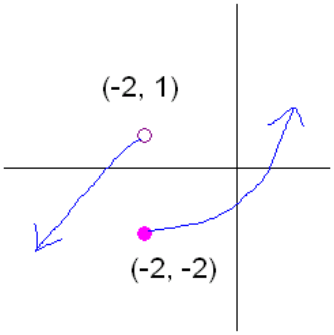
1. For the following graph of $f(x)$: Find a.) $f(1)$ and b.) $\lim_{x\to 1} f(x)$, if they exist.



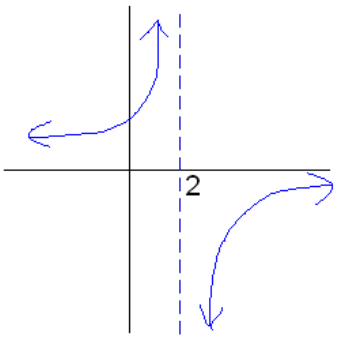
2. For the following graph of $f(x)$: Find a.) $f(2)$ and b.) $\lim_{x\to 2} f(x)$, if they exist.



3. For the following graph of $f(x)$: Find a.) $f(-2)$ and b.) $\lim_{x\to -2} f(x)$, if they exist.



4. For the following graph of $f(x)$: Find a.) $f(2)$ and b.) $\lim_{x\to 2} f(x)$, if they exist.



II. Find the following limits, if they exist.

5. $\lim_{x\to -2}\left(3x^2+5\right)$

6. $\lim_{x\to 1}\dfrac{x^2+2x+3}{x^2+1}$

7. $\lim_{x\to 2}\dfrac{x-2}{x^2-4}$

8. $\lim_{x\to 1/3}\dfrac{x^2-1/9}{3x-1}$

9. $\lim_{x\to -5} \dfrac{3x^2+13x-10}{2x^2+11x+5}$

10. $\lim_{h\to 0} \dfrac{2x^2-2(x+h)^2}{h}$

11. $\lim_{h\to 0} \dfrac{20000+3h}{h}$

12. $\lim_{x\to \infty} \dfrac{5-2x^3}{x^3-4x}$

13. $\lim_{x\to \infty} \dfrac{x^2+3x}{4x^3-1}$

14. $\lim_{x\to -\infty} \dfrac{x^2-5x+2}{x-2}$

III.

15. In problems 1 through 4, state any discontinuities, and also state which condition of continuity is not satisfied.

16. Suppose $f(x)=\begin{cases} 2x+.30 & if\ 0<x\le 1 \\ -x+.50 & if\ 1<x<2 \end{cases}$ Is $f(x)$ continuous at x = 1?

17. Suppose $f(x)=\begin{cases} 2x+1 & if\ 1<x\le 2 \\ 3x-1 & if\ 2<x\le 3 \\ -x+5 & if\ 3<x\le 4 \end{cases}$ Is $f(x)$ continuous at x = 2? x = 3?

18. If $f(x)=\dfrac{x}{a-bx}$, where will the function be discontinuous?

19. If $f(x)=\dfrac{x^2-7x-10}{(x-5)(x^2-4)}$, where will $f(x)$ de discontinuous?

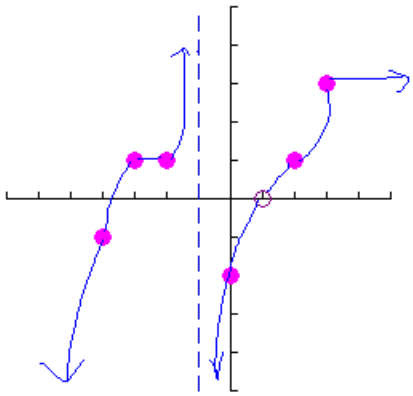
IV.

20. Find the instantaneous rate of change of $f(x)$ if $f(x)=3x^2-1$ by using the limit definition.

21. Write the equation of the tangent line to the graph of $f(x)=3x^2-4x$ at the point (2, 4).

22. For a certain product of the total revenue is $R(x) = 250x - .05x^2$. Find the marginal revenue when x = 100. Interpret your result.

23. For all of the integer values -4, -3, -2, -1, 0, 1, 2, 3 determine if the function is continuous and/or differentiable for the graph below.



V. Find y' for the following:

24. $y = 4x^{-3}$

25. $y = 3x^3 - \frac{1}{\sqrt{x}} + \frac{3}{x^2}$

26. $y = -\frac{14}{x^3} + \frac{3}{2\sqrt{x}} + \frac{2}{\sqrt[3]{x^5}}$

VI.

27. At what value(s) of x does the slope of the tangent to $y = x^3 - 2x^2 + 5x - 16$ equal 4.

28. A manufacturer determined that the profit from selling x units is $P(x) = .003x^2 + 10x$. Find the marginal profit $P(x)$, when 50 units are sold. Explain the result.

VII. Find $f'(x)$ for the following:

29. $f(x)=\left(x^3+4x^2+1\right)\left(x^3-1\right)$

30. $f(x)=\dfrac{5x^2-2x+1}{2x+1}$

VIII.

31. Write the equation of the tangent line to $y=\dfrac{x+2}{x+1}$ at $\left(1,3/2\right)$.

32. Suppose the demand for a product is given by $q=\dfrac{50,000}{p^2+10p+25}$ where q = # units demanded and p = price per unit. Find the rate of change in demand with respect to price when price = \$10. Interpret your answer.

VIII. Find $f'(x)$ for the following:

33. $f(x)=\left(2x^2+5x\right)^7$

34. $f(x)=2(1-3x)^4+9(1-x)^3+10(2x-1)^2+7$

35. $f(x)=\sqrt[3]{x^2+1}$

36. $f(x)=\dfrac{6}{\left(x^2+4x\right)^4}$

X.

37. The revenue function from selling a certain product is $R(x)=\dfrac{20x}{\sqrt{x-1}}$. Find the marginal revenue when 5 units are sold. Interpret your result.

Answers

I.

1. a.) undefined b.) 3
2. a.) 1 b.) -1
3. a.) -2 b.) does not exist
4. a.) undefined b.) does not exist

II.

5. 17
6. 3
7. $1/4$
8. $2/9$
9. $17/9$
10. -4x
11. does not exist
12. -2
13. 0
14. does not exist

III.

15. # 1 discontinuous at x = 1 because $f(1)$ is undefined

    # 2 is discontinuous at x = 2 because $\lim_{x\to 2} f(x) \neq f(2)$

    # 3 is discontinuous at x = -2 because $\lim_{x\to -2} f(x)$ does not exist.

    # 4 is discontinuous at x = 2 because $\lim_{x\to 2} f(x)$ and $f(2)$ does not exist.

16. discontinuous at x = 1
17. continuous at x = 2; discontinuous at x = 3.
18. $x = a/b$
19. x = 5, x = 2, x = -2

IV.

20. $f'(x) = 6x$
21. $y = 8x - 12$
22. $R'(100) = \$240$. The expected revenue by selling the next item (after 100) will be approximately \$240.
23.

x =	-4	-3	-2	-1	0	1	2	3
continuous?	Y	Y	Y	N	Y	N	Y	Y
differentiable?	Y	N	N	N	Y	N	Y	N

V.

24. $y' = -12/x^4$
25. $y' = 9x^2 + \dfrac{1}{2x^{3/2}} - \dfrac{6}{x^3}$
26. $y' = \dfrac{42}{x^4} - \dfrac{3}{4x^{3/2}} - \dfrac{10}{3x^{8/3}}$

VI.

27. x = $\frac{1}{3}$, x = 1

28. $P'(50) = \$10.30$ $P'(50) \Rightarrow$ If the number if units sold increases to 51, the profit will increase by \$10.30.

VII.

29. $f'(x) = \left(3x^2 + 8x\right)\left(x^3 - 1\right) + \left(x^3 + 4x^2 + 1\right)\left(3x^2\right)$

30. $f'(x) = \dfrac{(2x+1)(10x-2) - \left(5x^2 - 2x + 1\right)(2)}{(2x+1)^2}$

VIII.

31. $f'(1) = -\frac{1}{4}, \quad y - \frac{3}{2} = -\frac{1}{4}(x-1) \rightarrow y = -\frac{1}{4}x + \frac{7}{4}$

32. $q' = \dfrac{-50,000(2p+10)}{\left(p^2 + 10p + 25\right)^2}$ $q'(10) = \dfrac{-50,000(30)}{(225)^2} = -29.63$ If the price increases by \$1 to \$11, demand will decrease by about 30 units.

IX.

33. $f'(x) = 7\left(2x^2 + 5x\right)^6 (4x+5)$

34. $f'(x) = 8(1-3x)^3(-3) + 27(1-x)^2(-1) + 20(2x-1)(2)$

35. $f'(x) = \frac{1}{3}\left(x^2+1\right)^{-2/3}(2x)$

36. $f'(x) = -24\left(x^2 + 4x\right)^{-5}(2x+4)$

X.

37. $R'(x) = \dfrac{\sqrt{x-1}(20) - 20x\left[\frac{1}{2}(x-1)^{-1/2}(1)\right]}{\left(\sqrt{x-1}\right)^2}$

$$R'(5) = \frac{\sqrt{5-1}(20) - 20(5)\left[\frac{1}{2}(5-1)^{-1/2}(1)\right]}{\left(\sqrt{5-1}\right)^2}$$

$$= \frac{2(20) - 100\left(\frac{1}{2}\cdot\frac{1}{2}(1)\right)}{2^2} = \frac{40-25}{4} = \frac{15}{4} = \$3.75$$

If one more unit is sold (for a total of 6 units) revenue will change by \$3.75