Math 2414
Calculus II
Lab Exercise # 3

Name _____ Date _____

Section # _____ Semester _____

Attach the computer printouts to this sheet.

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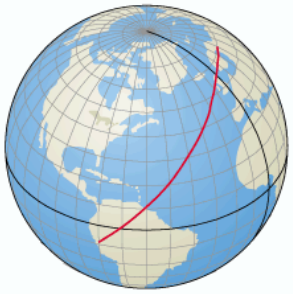
1. Summation notation will be used for i = 1 to n.
2. $\int_0^a R * \sec(\phi) d\phi$
3. run the integral for each.
4. no help necessary.

Making a Mercator Map

When flying or sailing, pilots expect to be given a steady compass course to follow. On a standard flat map, this is difficult because a steady compass course results in a curved line, as shown in the figures below.



Globe: flight with constant 45° bearing

Standard flat map: flight with constant 45° bearing

Mercator map: flight with constant 45° bearing

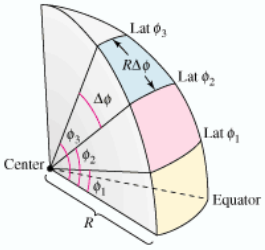
For curved lines to appear as straight lines on a flat map, Flemish geographer Gerardus Mercator (1512–1594) realized that latitude lines must be stretched horizontally by a scaling factor of sec ϕ, where ϕ is the angle of the latitude line. For the map to preserve the angles between latitude and longitude lines, the lengths of longitude lines are also stretched by a scaling factor of sec ϕ at latitude ϕ. The Mercator map has latitude lines that are not equidistant, as shown in the third figure above.

To calculate these vertical lengths, imagine a globe with latitude lines marked at angles of every $\Delta\phi$ radians, with $\Delta\phi = \phi_i - \phi_{i-1}$. The arc length of consecutive latitude lines is $R\Delta\phi$. On the Mercator map, the vertical distance between the equator and the first latitude line is $R\Delta\phi \sec \phi_1$. The vertical distance between the first and second latitude lines is $R\Delta\phi \sec \phi_2$. The vertical distance between the second and third latitude lines is $R\Delta\phi \sec \phi_3$, and so on, as shown in the figure on the right below.

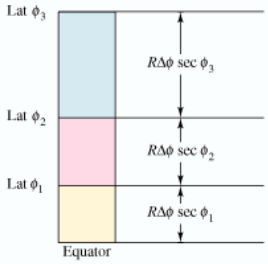
On a globe, the angle between consecutive latitude lines is $\Delta\phi$, and the arc length between them is $R\Delta\phi$ (see the left-hand figure below). On a Mercator map, the vertical distance between the ith and $(i-1)$st latitude lines is $R\Delta\phi \sec \phi_i$, and the distance from the equator to the ith latitude line is approximately

$$R\Delta\phi \sec \phi_1 + R\Delta\phi \sec \phi_2 + \cdots + R\Delta\phi \sec \phi_i$$

(see right-hand figure below).



Globe



Mercator map

QUESTIONS

1. Use summation notation to write an expression to calculate how far from the equator to draw the line representing latitude ϕ_n.
2. In the calculations above, Mercator realized that the smaller the value used for $\Delta\phi$, the better the map became (better in the sense that straight lines could be used to plot steady compass courses). From your knowledge of calculus, how could you use Mercator's observation to calculate the total vertical distance of a latitude line from the equator?
3. Use the result of Question 2 to find how far from the equator to place latitude lines whose angles are 10°, 20°, 30°, 40°, and 50°. (Use a globe radius of $R = 6$ inches.)
4. What problem do you encounter when you attempt to calculate how far from the equator to place the North Pole?

The concepts presented here will be explored further in this chapter. For an extension of this application, select the Calculus Lab button.

Calculus Lab