Chapter 8 Notes (FOR EXAM 5)

Sect. 8.1 Sequences and Summation Notation

a. An infinite sequence $\{a_n\}$ is a function whose domain is the set of positive integers. The function values, or terms, are represented by $a_1, a_2, a_3, ..., a_n, ...$
b. Sequences can be defined using recursion formulas that define the nth term as a function of the previous term.
c. Factorial notation: $n! = n(n-1)(n-2)(n-3)...(3)(2)(1)$
d. Summation notation: $\sum a_i = a_1 + a_2 + a_3 + ... + a_n$

Sect. 8.2 Arithmetic Sequences

A. In an arithmetic sequence , each term after the first differs from the preceding term by a constant, the common difference. Subtract any term from the term that directly follows to find the common difference.
B. General term or nth term : $a_n = a_1 + (n-1)d$. The first term is a_1 and the common difference is d.
C. Sum of the first n terms : $S_n = \frac{n}{2}(a_1 + a_n)$

Sect. 8.3 Geometric Sequences

a. In a geometric sequence, each term after the first is obtained by multiplying the preceding term by a nonzero constant, the common ratio. Divide any term after the first by the term that directly precedes it to find the common ratio.
b. General term or nth term: $a_n = a_1 r^{n-1}$. The first term is a_1 and the common ratio is r.

c. Sum of the first n terms: $S_n = \frac{a_1(1-r^n)}{1-r}$, r does not equal 1.

d. An annuity is a sequence of equal payments made at equal time periods. The value of an annuity, A , is the sum of all deposits made plus all interest paid, given by

$$A = \frac{P\left[\left(1 + r/n\right)^{nt} - 1\right]}{r/n}$$

The deposit made at the end of each period is P, the annual interest rate is r compounded n times per year, and t is the number of years deposits have been made.

e. Sum of an infinite geometric series is $s = \frac{a_1}{1-r}; \quad -1 < r < 1$.

Sect. 8.4 Mathematical Induction

a. To prove that S_n is true for all positive integers n:
   1. Show that S_1 is true.
   2. Show that if S_k is true, then S_{k+1} is also true, for every positive integer k.

Sect. 8.5 The Binomial Theorem:

a. Binomial coefficient: $(n,r) = \frac{n!}{r!(n-r)!}$

b.Binomial Theorem: $(a+b)^n = (n,0)a^n + (n,1)a^{n-1}b + (n,2)a^{n-2}b^2 + ...(n,n)b^n$

c. The rth term in a binomial expansion: $(n,r-1)a^{n-r+1}b^{r-1}$